# Basic Math *Quick Reference* eBOOK

Click on **Contents** or **Index** in the left panel to locate a topic.

The math facts listed in this eBook are explained with Examples, Notes, Tips, & Illustrations in the **Basic Math Quick Reference Handbook**.

ISBN: 978-0-615-27390-7

**Peter J. Mitas**

*Printable Version of this eBook*


Quick Reference Handbooks

# Contents

Click a **CHAPTER TITLE** to jump to a page in the Contents:



… then click a **LINE IN THE CONTENTS** to jump to a topic.

# Whole Numbers

## Natural Numbers and Whole Numbers

- The numbers that we use to count are called natural numbers or counting numbers.
- The WHOLE NUMBERS include all the natural numbers and ZERO.

## Digits and Numerals

- The first ten whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called DIGITS. These digits are used to write every number.
- Written whole numbers are sometimes called NUMERALS.

## Place Value Notation

- PLACE VALUE NOTATION allows the position of a digit in a number to determine its value. Each place for a digit in a whole number represents ten times the place to its right.

| THOUSANDS | HUNDREDS | TENS | ONES |
|---|---|---|---|
| 10 Hundreds | 10 Tens | 10 Ones | |

- The digit "0" (zero) is used to fill EMPTY PLACE VALUES.
- COMMAS are used in numerals to separate groups of three digits.
- A number is written in EXPANDED FORM when it is represented as the sum of the value of each of its digits.

## Rounding a Whole Number

- ROUNDING a whole number to a place value identifies which value it is closest to.
- A whole number can be rounded to any specified place value.

Steps

1. Begin by locating the digit in the place value to round to.
2. Focus on the digit to its right.
   - IF THE DIGIT TO ITS RIGHT IS LESS THAN FIVE replace that digit with a zero and then replace every digit to its right with a zero.
   - IF THE DIGIT TO ITS RIGHT IS FIVE OR GREATER replace the digit that is in the place value you are rounding to with the next higher digit, and then replace every digit to its right with a zero.

## Operations and Operators

- OPERATIONS are the common calculations performed on numbers: addition, subtraction, multiplication, and division.
- The symbols used to indicate an operation are called OPERATORS: +, –, x, and ÷.
- Placed between two numbers, an operator identifies the operation to be performed on those numbers.

| Operation | Operation with Operator |
|---|---|
| Addition | 12 + 3 = 15 |
| Subtraction | 12 – 3 = 9 |
| Multiplication | 12 × 3 = 36 |
| Division | 12 ÷ 3 = 4 |

# Adding Whole Numbers

- The result of adding numbers is called their SUM.
- Adding 0 to any number doesn't change its value.
- More than two digits can be added mentally.
- The order in which numbers are added doesn't matter.
- Add whole numbers column by column, starting with the ones column.

  Steps

  1. Line up digits having the same place value in the same column.
  2. Add the digits in the ones column. Repeat with the remaining columns, moving left one column at a time.

     - IF THE SUM OF A COLUMN REACHES 10 OR MORE, **CARRY** the digit in the tens place of this sum into the next larger place value.

# Subtracting Whole Numbers

- The result of subtracting one number from another is called their DIFFERENCE.

- Subtracting 0 from a number doesn't change its value.

- Subtracting a number from itself results in 0.

- Subtract whole numbers column by column, starting with the ones column.

  Steps

  1. Line up digits having the same place value in the same column, writing the larger number above the smaller one.

  2. Subtract the digits in the ones column. Repeat with the remaining columns, moving left one column at a time.

     - IF THE UPPER DIGIT IS LESS THAN THE DIGIT BEING SUBTRACTED, **BORROW** 10 from the next place value on the left.

- Check the answer to a subtraction problem by mentally adding up from the answer to see if the result is the top number.

# Multiplying Whole Numbers

- Multiplication is repeated addition.
- The result of multiplying numbers is called their PRODUCT.
  There are several ways to indicate multiplication.
  12 times 3 can be written as:
  $12 \times 3$, $12 \cdot 3$, 12 * 3, 12(3), (12)(3)
- Multiplying a number by 1 doesn't change its value.
- Multiplying a number by 0 results in 0.
- To find the product of three numbers, first multiply two of them. Then, multiply the result by the third number.
- The order in which numbers are multiplied doesn't matter.
- Multiply whole numbers column by column, starting with the ones column.

  Steps

  1. Line up digits having the same place value in the same column, writing the number with the most digits above the number with the least number of digits.

  2. Multiply the top number by the digit in the ones column of the bottom number.

     - IF THE BOTTOM NUMBER HAS MORE THAN ONE DIGIT, multiply the top number by each digit, beginning to write each result under the multiplying digit.
       Then add the products together.

# Dividing Whole Numbers

- The result of dividing one number by another is called their QUOTIENT.
- The number being divided is called the DIVIDEND.
- The number being divided *into* the dividend is called the DIVISOR.
- There are several ways to indicate division.

  12 divided by 3 can be written as:

  $12 \div 3,\ \frac{12}{3},\ 12/3,\ 3\overline{)12}$

- Dividing any number by 1 doesn't change its value.
- Dividing 0 by any number other than 0 results in zero.
- DIVIDING BY ZERO is not permitted.
- Dividing whole numbers could leave a REMAINDER.
- Check the answer to a division problem by multiplying the quotient by the divisor and then adding any remainder. The result should equal the dividend.
- There are two ways to divide one number by another: short division and long division.
- Use SHORT DIVISION to divide any whole number by a single digit.
- Use LONG DIVISION to divide any whole number by a number that has two or more digits.
- If it is hard to see how many times a large divisor can go into a number, mentally round the numbers.

# Divisibility Rules

- A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.
- A number divisible by 2 is an EVEN NUMBER.
  A number that is not divisible by 2 is an ODD NUMBER.
- A number is divisible by 10 if its last digit is 0.
- A number is divisible by 5 if its last digit is 0 or 5.
- A number is divisible by 4 if its last two digits are divisible by 4.
- A number is divisible by 8 if its last three digits are divisible by 8.
- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 9 if the sum of its digits is divisible by 9.

## Multiples of a Whole Number

- A MULTIPLE of a whole number is any number that it divides exactly.
- When a number is a multiple of several whole numbers, it is called a COMMON MULTIPLE of the numbers.
- The LEAST COMMON MULTIPLE of several whole numbers is called their LCM.
- Every group of whole numbers has an LCM.

## Factors of a Whole Number

- A FACTOR of a whole number is a number that exactly divides it. Every whole number has a limited number of factors.
- When a number is a factor of several whole numbers it is called a COMMON FACTOR.
- The GREATEST COMMON FACTOR of several whole numbers is called their GCF.
- Every group of whole numbers has a GCF.

# Prime and Composite Numbers

- Any whole number that has only two factors, 1 and itself, is a PRIME NUMBER.
- 0 and 1 are not prime numbers.
- 2 is the lowest prime number and is the only *even* prime number.
- When 1 is the only common factor of several numbers, they are said to be RELATIVELY PRIME.
- A COMPOSITE NUMBER is a whole number greater than 2 that is not prime.
- Every composite number is the product of prime numbers.
- Use a PRIME FACTOR TREE to find the prime factors of a composite number.
- Prime factor trees can be used to find both the greatest common factor and least common multiple of several numbers.

# Fractions

## Fraction, Numerator, and Denominator

- A FRACTION represents one or more equal parts of a whole.
- The number above the line in a fraction is called the NUMERATOR of the fraction. The number below the line in a fraction is called the DENOMINATOR of the fraction.
- When the numerator and denominator of a fraction are equal, the fraction equals 1.
- There are as many ways to write 1 as a fraction as there are numbers.
- Fractions that have the same denominator are said to have a COMMON DENOMINATOR.

## Fractions as Division

- A fraction can be treated as a division statement.
- Any REMAINDER to a division problem should be written as a fraction having a denominator equal to the divisor.

## Proper & Improper Fractions; Mixed Numbers

- When a fraction has a numerator that is less than the denominator, its value is less than 1 and it is called a PROPER FRACTION.
- When a fraction has a numerator that is *not* less than the denominator, its value is greater than or equal to 1 and it is called an IMPROPER FRACTION.
- A MIXED NUMBER is a number that has a whole number in addition to a proper fraction.

## Fractional Equivalence and Conversions

### Equal fractions

- When two different fractions represent the same quantity, they are equivalent, or EQUAL FRACTIONS.
- Multiplying or dividing both numerator and denominator of a fraction by the same number results in an equivalent fraction.
- When two fractions are equal, their CROSS-PRODUCTS are equal.

### Writing a fraction in lowest terms

- A fraction is in SIMPLEST FORM (or REDUCED TO LOWEST TERMS) when 1 is the only common factor of its numerator and denominator.
- To reduce a fraction to lowest terms, divide its numerator and denominator by their greatest common factor (GCF).

(***Fractional Equivalence and Conversions*** *Continues)*

## Writing a fraction in higher terms

- Any fraction can be written as another fraction that has a greater numerator and denominator.

  Steps

  1. Divide the new denominator by the denominator of the original fraction.
  2. Multiply this number by the numerator of the original fraction and write the result as the numerator of the new fraction.

## Equivalent improper fractions, mixed numbers & whole numbers

- Every improper fraction is equal to either a whole number or mixed number.

- Every mixed number is equal to an improper fraction.

## Writing an improper fraction as an equivalent whole number or mixed number

- Divide the denominator into the numerator. Write the resulting whole number. Any remainder becomes the numerator of a fraction in a mixed number.

- An improper fraction is written in SIMPLEST FORM when it is converted to an equivalent mixed number or whole number.

## Writing a whole number as an equivalent improper fraction

- To write a whole number as an improper fraction, either write a 1 as its denominator, or write it in higher terms.

*(**Fractional Equivalence and Conversions** Continues)*

## Writing a mixed number as an equivalent improper fraction

- Since a mixed number consists of a whole number and a fraction, convert the whole number into a fraction and then add it to the existing one.

  Steps

  1. Make the denominator of the improper fraction the same as that in the fraction of the mixed number.
  2. To calculate the numerator of the improper fraction:
     (1) Multiply the denominator of the fraction by the whole number to determine the number of parts in the wholes.
     (2) Add this number to the numerator of the fraction in the mixed number.

## Replacing several fractions with equal fractions having their least common denominator

- The LEAST COMMON DENOMINATOR (LCD) of several fractions is found by calculating the least common multiple (LCM) of their denominators. The LCD is then used as the denominator of equal fractions written in higher terms.

# Ordering Fractions

- To order fractions that have a common denominator, write them in the order of their numerators.
- To order fractions that have different denominators, first replace them with equivalent fractions that have a common denominator. Then, order the fractions that have a common denominator.

  Steps

  1. Find the least common denominator of the fractions.
  2. Replace the fractions with equivalent fractions that have the common denominator.
  3. Write the equivalent fractions in the order of their numerators.
  4. Write the original fractions in order.

# Adding Numbers That Include Fractions

## Adding fractions that have a common denominator

- Fractions can be added only when they have a common denominator.

  Steps

  1. Begin by writing the fractions beneath one another.
  2. Add the numerators of the fractions and use the sum as the numerator of the result.
  3. Use the denominator of the fractions being added as the denominator of the result.

(***Adding Numbers That Include Fractions*** *Continues)*

## Adding numbers that include fractions, whole numbers, and mixed numbers

- Adding numbers that include fractions, whole numbers, and mixed numbers is similar to adding fractions. The only difference is that the whole numbers are lined up in another column and added separately.

  Steps

  1. Begin by writing the numbers beneath one another.
  2. Add the fractions.
  3. Add the whole numbers.

## Adding fractions that do not have a common denominator

- When fractions have different denominators, replace them with equivalent fractions that have the least common denominator. Then add as usual.

  Steps

  1. Begin by writing the numbers beneath one another.
  2. Find the least common denominator.
  3. Write the fractions in higher terms, using the least common denominator in the equivalent fractions.
  4. Add the numbers whose fractions now have a common denominator.

*(**Adding Numbers That Include Fractions** Continues)*

## Writing the answer to an addition problem in simplest terms

- After adding fractions, if the result contains a fraction that is not in lowest terms, reduce the fraction to lowest terms.

- After adding fractions, if the result contains an improper fraction, replace it with an equivalent whole number or mixed number.
  - IF THE RESULT ALREADY INCLUDES A WHOLE NUMBER, add it to the whole number or mixed number that replaced the improper fraction.

- After adding fractions, if the result contains an improper fraction that is not in lowest terms, change it to a mixed number and reduce the fraction.

# Subtracting Numbers That Include Fractions

## Subtracting fractions that have a common denominator

- Fractions can be subtracted only when they have a common denominator.

  Steps

  1. Begin by writing the smaller fraction beneath the larger one.
  2. Subtract the numerator of the bottom fraction from the numerator of the top fraction. Use this answer as the numerator of the result.
  3. Use the denominator of the fractions being subtracted as the denominator of the result.

## Subtracting numbers that include fractions, whole numbers, and mixed numbers

- Subtracting numbers that include fractions, whole numbers, and mixed numbers is similar to subtracting fractions. The only difference is that the whole numbers are lined up in another column and subtracted separately.

  Steps

  1. Begin by writing the smaller number beneath the larger one.
  2. Subtract the fractions.
  3. Subtract the whole numbers.

*(**Subtracting Numbers That Include Fractions** Continues)*

## Subtracting fractions that do not have a common denominator

- When fractions have different denominators, replace them with equivalent fractions that have a common denominator. Then subtract as usual.

  Steps

  1. Begin by writing the smaller number beneath the larger one.
  2. Find the least common denominator.
  3. Write the fractions in higher terms, using the least common denominator in the equivalent fractions.
  4. Subtract the numbers whose fractions now have a common denominator.

## Subtracting fractions when the fraction to be subtracted is greater than the fraction above it

- If there is no fraction in the top number, only a whole number, borrow 1 from it. Write the 1 as a fraction with numerator and denominator equal to the denominator of the fraction being subtracted.

- If the top number has a fraction, that fraction must be added to the 1 you borrowed.

## Writing the answer to a subtraction problem in simplest terms

- After subtracting fractions, if the result contains a fraction that is not in lowest terms, reduce it to lowest terms.

# Multiplying Numbers That Include Fractions

## Multiplying fractions

- When multiplying fractions, write them in a single row, and then multiply straight across.

  Steps

  1. Write the fractions in a single row separated by multiplication operators.
  2. Multiply the numerators together.
  3. Multiply the denominators together.

- More than two fractions can be multiplied together at once.

## Multiplying numbers that include fractions and whole numbers

- When you multiply a fraction by a whole number, the whole number only multiplies the numerator of the fraction.

  Steps

  1. Write a 1 under each whole number to convert it to an improper fraction.
  2. Multiply the numerators together.
  3. Multiply the denominators together.

*(**Multiplying Numbers That Include Fractions** Continues)*

## Multiplying numbers that include fractions and mixed numbers

- When a multiplication problem includes mixed numbers, replace them with equivalent improper fractions and then multiply.

  Steps

  1. Replace each mixed number with an equivalent improper fraction.
  2. Multiply the numerators together.
  3. Multiply the denominators together.

## Writing the answer to a multiplication problem in simplest terms

- Reduce a fraction that is not in lowest terms and replace an improper fraction with an equivalent whole number or mixed number.

## Canceling

- Remove factors common to numerators and denominators before multiplying. This simplifies the problem and forces the answer to be in lowest terms.

## Reciprocal fractions

- Two fractions, in which the numerator of each fraction equals the denominator of the other, are called RECIPROCALS of one another.

- The product of two reciprocals is 1.

# Dividing Numbers That Include Fractions

## Multiplying instead of dividing

- When dividing numbers that include fractions, replace the divisor by its reciprocal and change the problem to one of multiplication. Then follow the procedure for multiplying fractions together.
  Steps
  1. Write the numbers in a single row as **(dividend) ÷ (divisor)**.
  2. Replace each whole and mixed number with an equivalent improper fraction.
  3. Change the division operator (÷) to multiplication (×) and replace the divisor with its reciprocal.
  4. Multiply the fractions together.
  5. Simplify the answer. Reduce a fraction that is not in lowest terms and replace an improper fraction with an equivalent whole number or mixed number.

- The procedure for dividing fractions also works with whole numbers. We don't use it to divide whole numbers because short and long division are much more efficient.

## Why multiply when dividing fractions

- Instead of needing another procedure for dividing fractions, we reuse the one for multiplying fractions because it provides the correct answer.

## Complex fractions

- COMPLEX FRACTIONS are fractions that have a fraction in the numerator and/or the denominator.

- Simplify a complex fraction by treating it as a division statement.

# Comparing Operations on Fractions

| | Addition | Subtraction | Multiplication | Division |
|---|---|---|---|---|
| | Two or more numbers | Only two numbers | Two or more numbers | Only two numbers |
| **SETUP** | Write the numbers in columns. | | Write the numbers in a single row. | |
| | DENOMINATORS MUST BE EQUAL.<br>If they are not, find the least common denominator of the fractions. Then replace the fractions with equivalent fractions that have the common denominator. | | ALL TERMS MUST BE PROPER OR IMPROPER FRACTIONS.<br>If there are any mixed or whole numbers, write them as improper fractions. | |
| | | THE BOTTOM FRACTION MUST BE LESS THAN THE TOP ONE.<br>If not, borrow 1 from the whole number on top. | | Change "÷" to "×" and replace the divisor with its reciprocal (swap the numerator and denominator). |
| **SOLVE** | | | Cancel common factors from the numerators and denominators. | |
| | Add numerators.<br>Copy the denominator. | Subtract numerators.<br>Copy the denominator. | Multiply numerators.<br>Multiply denominators. | |
| | Add whole number terms. | Subtract whole number terms. | | |
| **SIMPLIFY** | ANY FRACTION IN THE RESULT MUST BE IN LOWEST TERMS.<br>If needed, reduce the fraction to lowest terms.<br><br>ANY FRACTION IN THE RESULT MUST BE PROPER.<br>If the result has an improper fraction, change the improper fraction to a mixed or whole number. | | | |

# Decimal Numbers

## Representing Wholes and Parts as Decimal Numbers

- A decimal number is a number that has a decimal point separating a quantity of wholes from a quantity of parts.
- The place values for decimal numbers increase by a multiple of 10. Each digit in a decimal number is a multiple of the place value it occupies.
- Decimal place values are symmetric about the ones place and not symmetric about the decimal point. The decimal point merely separates the wholes from the parts. (There is no “oneths” place.)
- Every whole number is a decimal number. Even though a whole number has no decimal point, the point can be added to the right of the ones column whenever whole numbers appear in a problem that includes numbers that have decimal points.
- Decimal numbers are like mixed numbers because they can include both wholes and parts of a whole.
- When reading a decimal numeral that includes a whole and parts of a whole, the decimal point is read as “and”. This is the same as saying “and” when reading a mixed number.
- When reading the part in a decimal number, only the place value of the rightmost digit is mentioned.

## Rounding a Decimal Number to a Specific Place Value

- The steps for rounding a decimal number are similar to those for rounding a whole number.

  Steps

  1. Begin by locating the digit in the place value to round to.
  2. Focus on the digit to its right.
     - ONLY IF THE DIGIT TO ITS RIGHT IS 5 OR GREATER, add 1 to the digit in the place value you are rounding to.
  3. Focus on whether the place value you are rounding to is to the left or right of the decimal point.
     - IF THE PLACE VALUE YOU ARE ROUNDING TO IS TO THE LEFT OF THE DECIMAL POINT, replace digits up to the decimal point with zeros and discard the decimal point and all digits to its right.
     - IF THE PLACE VALUE YOU ARE ROUNDING TO IS TO THE RIGHT OF THE DECIMAL POINT, discard all digits to its right.

## Ordering Decimal Numbers

- When ordering decimal numbers it helps to give them each the same number of digits to the right of the decimal point by writing additional zeros if necessary.

# Types of Decimal Numbers

- When a decimal number has a final digit it is called a TERMINATING decimal number.

- When the digits in a decimal number go on forever, it is called a NON-TERMINATING decimal number.
  Three dots (…) are written after it to indicate that it goes on forever.

- When the digits in a non-terminating decimal number repeat, it is called a NON-TERMINATING, REPEATING decimal number.
  A line is written over the repeating digits to indicate that they repeat.

- When the digits in a non-terminating decimal number do not repeat, it is called a NON-TERMINATING, NON-REPEATING decimal number.

## Adding Decimal Numbers

- The most important thing to do when adding decimal numbers is to line up the decimal points.

  Steps

  1. Write a decimal point to the right of any whole number.
  2. Write the numbers beneath one another with their decimal points lined up. (This forces the place values to line up.)
  3. Fill-in zeros to make each number have the same number of places to the right of the decimal point.
  4. Add the numbers as if there were no decimal points.
  5. Place a decimal point in the answer under the other decimal points.

## Subtracting Decimal Numbers

- The most important thing to do when subtracting decimal numbers is to line up the decimal points.

  Steps

  1. Write a decimal point to the right of any whole number.
  2. Write the smaller number beneath the larger number with their decimal points lined up. (This forces the place values to line up.)
  3. Fill-in zeros to make each number have the same number of places to the right of the decimal point.
  4. Subtract the numbers as if there were no decimal points.
  5. Place a decimal point in the answer under the other decimal points.

## Multiplying Decimal Numbers

- The most important thing to do when multiplying decimal numbers is to place the decimal point correctly in the answer.

  Steps

  1. Write the numbers beneath one another with their rightmost digits beneath one another.
  2. Multiply as if there were no decimal point.
  3. When you get the result, count the number of digits to the right of the decimal points in the numbers that were multiplied.
  4. Place the decimal point in the answer so that the answer has the same number of digits to the right of the point as the count you made in Step 3.

- If one of the numbers being multiplied is a whole number, it will not have a decimal point. The implied decimal point is to its right and there are no digits following it.

## Dividing Decimal Numbers

- To divide decimal numbers, focus on the decimal points in the divisor, the dividend, and the quotient.

- We move the decimal point in the divisor and the dividend before we start to divide in order to correctly place the decimal point in the quotient.

- Don’t stop if the answer has a remainder. Add more zeros to the dividend until your answer has one digit more than the place value you are rounding to.

# Decimal Numbers and Equivalent Fractions

## Converting a fraction to a decimal number

- To convert a proper fraction to a decimal number, divide the denominator into the numerator.

## Converting a mixed number to a decimal number

- To convert a mixed number to a decimal number, copy the whole number that is in the mixed number, and then convert the fraction to a decimal number.

## Converting a terminating decimal number to a fraction

- The words we say when properly reading a decimal number can simply be written as a fraction or a mixed number.

## Converting a repeating decimal number to a fraction

- To convert a non-terminating, repeating decimal number to a fraction, place the repeating digits over just as many 9s.
- To convert a repeating decimal that starts with non-repeating digits, first isolate the repeating digits.

## Ordering fractions and decimal numbers

- To order numbers that include both fractions and decimal numbers, first convert the fractions to decimal numbers, and then order the decimal numbers.

# Universal Number Concepts

THE CONCEPTS DESCRIBED IN THIS SECTION APPLY TO ALL NUMBERS, INCLUDING WHOLE NUMBERS, FRACTIONS, DECIMAL NUMBERS, AND POSITIVE AND NEGATIVE NUMBERS.

## Set Notation

- A notation is a way of writing something in a standard way. SET NOTATION uses curly braces { } to surround the group of items that make up a set.

## Powers

### Powers of a number

- When several identical numbers are multiplied together the result is called a POWER of that number.
- The same number could be a power of more than one number.
- There are several ways to verbally express the power of a number.
- The NUMBER OF A POWER identifies how many base numbers are multiplied together.
- The number of a power of 10 is equal to the number of zeros in the power of 10.

*(**Powers** Continues)*

## Squares, square roots, cubes, and cube roots

- The 2nd power of a number is called its SQUARE.
- The number that is multiplied by itself to result in a square is called the SQUARE ROOT of the result. The notation for square root is a RADICAL SYMBOL $\sqrt{\ }$.
- Most square roots are non-repeating, non-terminating decimal numbers.
- The 3rd power of a number is called its CUBE.
- When three identical numbers are multiplied to result in a cube, the number is called the CUBE ROOT of the result. The notation for cube root is a radical symbol preceded by a small 3.

## Multiplying by powers of 10

- To multiply a decimal number by a power of 10, move the decimal point to the right as many places as there are zeros in the power of 10.
- When the point must move past a place that doesn't have a digit, write a zero in that place.

## Dividing by powers of 10

- To divide a decimal number by a power of 10, move the decimal point to the left as many places as there are zeros in the power of 10.
- When the point must move past a place that doesn't have a digit, write a zero in that place.

# Exponential Notation

## Exponentials

- EXPONENTIAL NOTATION is a simplified way to write a power of a number.
- Each symbol used in exponential notation has a name.

  $2^7$ is called an EXPONENTIAL.

  2 is the BASE of the exponential.

  7 is the EXPONENT.
- To find the VALUE OF AN EXPONENTIAL, multiply as many base numbers together as indicated by the exponent.
- An exponent of 1 indicates that the value of the exponential equals its base.
- When no exponent is written, the implied exponent is 1.

## Simplifying exponential products and quotients

- Simplify the product or quotient of exponentials whenever the numeric value of the product does not have to be precisely known, or when finding the value would require excessive calculation.
- Add exponents to simplify the product of exponentials that have the same base.
- To simplify a POWER OF A POWER, multiply exponents.
- Subtract exponents to simplify the quotient of exponentials that have the same base.

*(**Exponential Notation** Continues)*

## Writing prime factors in exponential notation

- Because a composite number often contains several identical prime factors, exponential notation is commonly used to describe the prime factors of a whole number.

## Multiplying and dividing by powers of 10 written in exponential notation

- To multiply a decimal number by a power of 10 written in exponential notation, move the decimal point to the right as many places as the exponent of the power of 10.

- To divide a decimal number by a power of 10 written in exponential notation, move the decimal point to the left as many places as the exponent of the power of 10.

## Scientific notation (for large numbers)

- SCIENTIFIC NOTATION is the way scientists write approximations for very large numbers.

- Scientific notation is the product of two terms. The first term is a number between 1 and 10. The second term is a power of 10.

- Any large number can be written in scientific notation.

  Steps

  1. Copy the digits, ignoring the zeros at the end of the number.
  2. Place a decimal point after the first digit in the number that you wrote.
  3. Skip the first digit in the original number and count the remaining digits.
  4. Use this number as the exponent in the power of 10.

## An exponent of 0

- Zero can be an exponent.

- An exponent of 0 indicates that the *value* of the exponential is 1.

*(**Exponential Notation** Continues)*

## Fractional exponents

- An exponent of $\frac{1}{2}$ indicates that the exponential represents a square root.

## Powers of fractions and decimal numbers

- Powers of fractions can be written in exponential notation.
- Powers of decimal numbers can be written in exponential notation.
- A power of a number can be less than its base.

# Order of Operations

- Operations that should be performed together are grouped by parentheses. Operations within parentheses should be performed before other operations.

- To simplify expressions consisting of several numbers interspersed with operators and parentheses, a specific order of operations must be followed.

  Steps

  1. **Simplify expressions within PARENTHESES**

     Parentheses are usually written as: ( ).
     IF A PARENTHESIS IS WRITTEN INSIDE ANOTHER PARENTHESIS, the outer one is written as [ ] or { }.

     First simplify the numbers in the inside parenthesis, then simplify the numbers in the outer parenthesis.

  When all expressions within parentheses have been simplified:

  2. **Replace EXPONENTIALS with their values.**

  When all exponential numbers have been replaced by their values:

  3. **MULTIPLY and DIVIDE**
     Move left to right, performing multiplications and divisions as they are encountered.

  When the only remaining operations are addition and subtraction:

  4. **ADD and SUBTRACT**
     Move left to right, performing additions and subtractions as they are encountered.

- A common acronym used to remember the order of operations is PEMDAS.
  (“Please Excuse My Dear Aunt Sally.”)

# Properties of Numeric Operations

## Commutative property

- When the position of numbers can be changed in an expression without changing the result of an operation, the operation is said to have a COMMUTATIVE PROPERTY.

  Addition and multiplication both have commutative properties.

## Associative property

- When the order in which operations are performed does not affect the final result, the operation is said to have an ASSOCIATIVE PROPERTY.

- Addition and multiplication both have associative properties.

## Distributive property

- When an operation can be distributed over all the terms within a parenthesis, it is said to have a DISTRIBUTIVE PROPERTY.

- Multiplication is distributive over addition and subtraction.

## Inequality Symbols

- $\neq$ is the symbol for “NOT EQUAL TO”.
- $<$ is the symbol for “LESS THAN”.
- $>$ is the symbol for “GREATER THAN”.
- $\leq$ is the symbol for “LESS THAN OR EQUAL TO”.
- $\geq$ is the symbol for “GREATER THAN OR EQUAL TO”.
- Inequality symbols can be used to describe a range of numbers.

## Graphing Numbers on a Number Line

- The NUMBER LINE is a line on which every point corresponds to a number and every number to a point. It is divided into equal sections that represent a specific range of numbers.
- The number line allows us to visualize the order of numbers. A number to the right of any other number is the greater number.
- Individual numbers are graphed on the number line with dots.
- A set of continuous numbers is graphed on the number line as a thick line.

# Ratios, Proportions, and Percents

## Ratio

- A RATIO is a comparison of two quantities.
- A ratio is simplified the same way that a fraction is simplified.
- A ratio can be inverted and still be true.

## Proportion

- Two equal ratios form a PROPORTION.
- A proportion has equal cross-products.

## Percent

### What percent means

- ONE HUNDRED PERCENT (100%) of something represents all of it.
- ONE PERCENT (1%) of something represents $\frac{1}{100}$ of it.
  (1 percent literally means 1 per hundred.)
- Percents lets us compare different parts of a whole as if it were made up of 100 parts, no matter how many actual items make up the whole thing under consideration.

### Percents and equivalent decimal numbers

- A percent can be written as a decimal number by moving its decimal point two places to the left.
- A decimal number can be written as a percent by moving its decimal point two places to the right.

*(**Percent** Continues)*

## Percents and equivalent fractions

- To convert a percent to a fraction, place it over 100 and simplify the resulting fraction.
- To convert a fraction to a percent, first convert the fraction to a decimal number. Then convert the decimal number to a percent.

## Solving percent problems

- Every percent problem has the following format:
  PERCENT OF WHOLE = PART
  where one of these three terms will be missing.
- In a percent problem, “OF” means “TIMES”.
- Change the given format of a percent problem:
  PERCENT of WHOLE = PART
  into a format that is much more useful.
  DECIMAL NUMBER x WHOLE = PART
- To find the PART when you are given the percent and the whole, write the percent as a decimal number, and then multiply the decimal number by the whole.
- To find the WHOLE when you are given the percent and the part, write the percent as a decimal number, then divide the part by the decimal number.
- To find the PERCENT when you are given the whole and the part, divide the part by the whole, and then write the decimal answer as a percent.

## Percents greater than 100%

- Solve problems that involve percents greater than 100% with the same techniques used to solve other percent problems.

# Probability and Statistics
**(Selected Topics)**

## Probability of an Event

- The PROBABILITY of an event occurring is the ratio of favorable outcomes to possible outcomes.

$$\text{Probability} = \frac{\text{number of Favorable Outcomes}}{\text{number of Possible Outcomes}}$$

## Statistical Measures

- Numbers collected for statistics are frequently referred to as scores, or DATA SETS.
- The MEAN of a data set is its AVERAGE.
- To find the mean of a data set, first add the scores, and then divide their sum by the number of scores.
- The MODE of a data set is the most common score.
- A data set can have no mode, one mode, or more than one mode.
- The MEDIAN of a data set is the number in the middle of the scores after they are placed in order. (Half the scores are higher and half are lower.)
- When there is an odd number of scores, the median is the middle score in the ordered list.
- When there is an even number of scores, the median will be a number that is halfway between the two central scores in the ordered list.
- The RANGE of a data set is the difference between the highest score and the lowest score.

# Geometry and Measurement
**(Selected Topics)**

## Angles and Lines

### Line segment

- A LINE SEGMENT is a line connecting two points. The name of a line segment is written with a line over the names of its two end points.

### Angles

- An angle is formed when two straight lines intersect.
- A DEGREE is the unit of measurement of an angle.
- A $360^{o}$ angle is made by sweeping a line segment to form a circle.
- A $90^{o}$ angle is called a RIGHT ANGLE.
- A right angle is drawn with a small square at its center.
- An angle between $0^{o}$ and $90^{o}$ is called an ACUTE ANGLE.
- An angle greater than $90^{o}$ is called an OBTUSE ANGLE.
- A $180^{o}$ angle forms a straight line and is called a STRAIGHT ANGLE.

### Perpendicular and parallel lines

- PERPENDICULAR LINES are lines that intersect at a $90^{o}$ angle.
- PARALLEL LINES are lines that run in the same direction and never intersect one another.

# Polygons

## Planes and polygons

- In geometry, a flat surface is called a PLANE.
- When lines enclose a flat space in a plane they create a figure called a POLYGON. (A many-sided plane figure.)
- When the sides of a polygon are all equal to one another, the figure is called a REGULAR POLYGON.
- When all the sides of a polygon are not equal to one another, the figure is called an IRREGULAR POLYGON.

*(**Polygons** Continues)*

## Quadrilaterals

- A QUADRILATERAL is a polygon that has four sides.

| | |
|---|---|
| Rectangle | ▪ Irregular polygon<br>▪ Opposite sides are equal and parallel<br>▪ Four $90^o$ angles |
| Square | ▪ Regular polygon; all four sides are equal<br>▪ Opposite sides are parallel<br>▪ Four $90^o$ angles |
| Parallelogram | ▪ Irregular polygon<br>▪ Opposite sides are equal and parallel<br>▪ Opposite angles are equal |
| Rhombus | ▪ Regular parallelogram; all four sides are equal<br>▪ Opposite sides are parallel<br>▪ Opposite angles are equal |
| Trapezoid | ▪ Irregular polygon<br>▪ Two opposite sides are parallel |

*(**Polygons** Continues)*

## Triangles

- A triangle is a polygon that has three sides and three angles that add up to $180^{o}$.

| | |
|---|---|
| Right triangle | ▪ A triangle with one right angle<br>▪ The side opposite the right angle is called the HYPOTENUSE of the triangle. |
| Isosceles triangle | ▪ A triangle with two equal sides<br>▪ The two angles that are opposite the equal sides are equal to one another |
| Equilateral triangle | ▪ Regular polygon<br>▪ A triangle that has three equal sides<br>▪ Each angle is $60^{o}$ |
| Scalene triangle | ▪ A triangle that has three unequal sides |

## The Pythagorean Theorem

- The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.

# Circles

- The CIRCUMFERENCE of a circle is the length of the outside edge of the circle.
- The RADIUS of a circle is the distance from its center to its outside edge.
- The DIAMETER of a circle is the length of a line that cuts the circle in half. The diameter of a circle passes through the center of the circle, and is twice as long as the radius.
- The parts of any circle have fixed ratios.

$$\frac{\text{diameter}}{\text{radius}} = 2$$

$$\text{diameter} = 2 \times \text{radius}$$

The ratio of the diameter of a circle to the radius of the circle is 2 to 1.

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

$$\text{circumference} = \pi \times \text{diameter}$$

$$\text{circumference} = 2 \times \pi \times \text{radius}$$

The ratio of the circumference to the diameter is represented by the Greek letter $\pi$ ("**pi**").

- $\pi$ is a non-terminating, non-repeating decimal number.

$$\pi = 3.14159265358979323846264338327 95\ldots$$

- $\pi$ is approximated in whole number and decimal calculations as 3.14 or 3.1416.
- $\pi$ is approximated in fractional calculations as $3\frac{1}{7}$ or $\frac{22}{7}$.

# Length

- LENGTH is a measurement of distance.

- U.S. STANDARD UNITS OF LENGTH include: inches (in), feet (ft), yards (yd), and miles (mi).

  1 foot (ft) = 12 inches (in)
  1 yard (yd) = 3 feet = 36 inches
  1 mile (mi) = 5280 feet

- A DOUBLE QUOTATION MARK is sometimes used to indicate inches.

- A SINGLE QUOTATION MARK is sometimes used to indicate feet.

- METRIC UNITS OF LENGTH include: millimeters (mm), centimeters (cm), meters (m), and kilometers (km).

| | |
|---|---|
| 1 kilometer (km) | = 1000 meters |
| 1 meter (m) | = 100 centimeters = 1000 millimeters |
| 1 centimeter (cm) | = 10 millimeters = .01 meters |
| 1 millimeter (mm) | = .001 meters |

## Perimeter

- The PERIMETER of a polygon is the sum of the lengths of all of its sides.
- The perimeter of a rectangle is twice the sum of the length plus width.
- The perimeter of a square is four times the length of one side.
- The perimeter of a rhombus is four times the length of one side.
- The perimeter of an equilateral triangle is three times the length of one side.

## Area

- AREA is a measurement of a surface, and is measured in SQUARE UNITS.
- U.S. Standard units of area include: square inches ($in^2$), square feet ($ft^2$), square yards ($yd^2$), and square miles ($mi^2$).
- Metric units of area include: square centimeters ($cm^2$), square meters ($m^2$), and square kilometers ($km^2$).
- The area of a rectangle , parallelogram, or rhombus is base times height.
- The area of a square is the square of a side.
- The area of a triangle is half the base times height.
- The area of a circle is $\pi$ times the radius squared.

## Surface area

- SURFACE AREA is a measure of the total area of every surface of a solid object.

- A brick-shaped object is called a RECTANGULAR PRISM. The surface area of a rectangular prism is the sum of the areas of all six surfaces: top, bottom, front, back, left side, and right side.

## Volume

- VOLUME is a measure of occupied space. Volume is measured in cubic units.

- A CUBE is a rectangular prism having edges of equal length. The volume of a cube is length $\times$ width $\times$ height (or edge $\times$ edge $\times$ edge).

  The volume of a cube having a 1 inch edge is $1 \times 1 \times 1 = 1$ cubic inch.

- The volume of a rectangular prism is length $\times$ width $\times$ height.

# Positive and Negative Numbers

## Signed Numbers and the Number Line

- Numbers that are greater than a reference point are POSITIVE. Numbers that are less than a reference point are NEGATIVE.
- Positive numbers are written with a POSITIVE SIGN ( $^{+}$ ).
- Negative numbers are written with a NEGATIVE SIGN ( $^{-}$ ).
- Positive and negative numbers can be shown on a number line that has 0 as a reference point.
- 0 is called the ORIGIN.

## Sign and Absolute Value

- A positive or negative number is called a SIGNED NUMBER because it either has a positive sign or a negative sign.
- The sign tells us whether the number is greater than zero (positive) or less than zero (negative).
- Zero has no sign because it is the reference point.
- Any non-zero number that has no sign is positive.
- The numeric part of a signed number tells us how far from 0 the number is.
- The numeric term in a signed number is called its ABSOLUTE VALUE.
- The mathematical symbol for the absolute value of a number is two straight lines that sandwich the number between them.
- The absolute value of any number is always positive because it is the distance of that number from zero, regardless of whether it is to the right or left of zero.
- $|0| = 0$ because the distance of zero from zero (the reference point) is zero.

# Opposites

- The OPPOSITE of a signed number is a number with the same absolute value, but a different sign. That is, the number on the opposite side of zero that is the same distance from zero.
- The mathematical symbol for opposite is a dash.
- The opposite of 0 is 0.
- When there are several adjacent signs in front of an absolute value, they can all be replaced by a single positive or negative sign.

  Steps

  1. Count the dashes (opposite symbols and negative signs) that precede the absolute value of a number.
  2. Then …
     - IF THERE IS AN EVEN NUMBER OF DASHES, replace all symbols with a single positive sign.
     - IF THERE IS AN ODD NUMBER OF DASHES, replace all symbols with a single negative sign.

## Integers, Rational and Irrational Numbers, and Real Numbers

- Signed numbers are classified as integers, rational numbers, irrational numbers, and real numbers.
- The INTEGERS are all the whole numbers and their opposites.
- RATIONAL NUMBERS are numbers that can be written as fractions (ratios). They include all the integers, positive and negative fractions, terminating decimal numbers, and non-terminating repeating decimal numbers.
- IRRATIONAL NUMBERS are numbers that cannot be written as fractions. They include all non-terminating, non-repeating decimal numbers.
- The REAL NUMBERS include all rational and irrational numbers.

## Ordering Positive and Negative Numbers

- On a number line, just as with whole numbers, a number to the left of another number is the lesser number.
- Ordering rational numbers that are written as fractions or decimal numbers is the same as ordering integers: a number that is more negative is the lesser one.

## Adding Two Signed Numbers

- When two signed numbers that are being added have the same sign, add their absolute values. Give the answer the same sign as the numbers being added.
- When two signed numbers that are being added have different signs, subtract the smaller absolute value from the larger one. Give the answer the same sign as the number with the larger absolute value.
- Adding a signed number to its opposite results in 0.
- Use the same procedure when numbers have no signs. Treat unsigned numbers as positive numbers.
- Use the same procedure to add signed fractions or decimal numbers.

## Subtracting One Signed Number From Another

- Instead of subtracting a number, add its opposite.
- Adding the opposite of a number gives the same result as subtracting it.
- Convert every subtraction problem into one of addition, then follow the same procedure used to add signed numbers.

  Steps

  1. Replace the minus sign with a plus sign.
  2. Change the sign of the number being subtracted from positive to negative or negative to positive.
     (This replaces the number being subtracted with its opposite.)
  3. Add the two numbers, using the steps for adding signed numbers.

- Use the same procedure when numbers have no signs. Treat unsigned numbers as positive numbers.
- Use the same procedure to subtract signed fractions or decimal numbers.

# Expert Strategies for Adding and Subtracting Signed Numbers

- Treat all signs: plus, minus, positive, and negative signs as symbols.

  Steps

  1. Replace groups of adjacent symbols with a single symbol by counting the "–" symbols in the group.

     - IF THERE IS AN EVEN NUMBER OF "–" SYMBOLS IN THE GROUP, replace them all with a single "+" symbol.
     - IF THERE IS AN ODD NUMBER OF "–" SYMBOLS IN THE GROUP, replace them all with a single "–" symbol.

  2. When absolute values are separated by only one + or – symbol, note the symbols that are in front of each absolute value.

     - IF THE SYMBOLS ARE THE SAME, add the absolute values and give the answer the same symbol (sign).
     - IF THE SYMBOLS ARE DIFFERENT, subtract the smaller absolute value from the larger one, and give the answer the same symbol (sign) as the number with the larger absolute value.

- This technique works for all symbols that are grouped together, including opposite signs.

- When adding or subtracting several signed numbers, place the numbers into two groups: plus and minus.
  Then subtract the minus group from the plus group.

  Steps

  1. Replace groups of symbols with a single + or – symbol.
  2. Add the absolute values that are preceded by + symbols.
  3. Add the absolute values that are preceded by – symbols.
  4. Subtract the sum in Step 3 from the sum in Step 2.

# Multiplying Signed Numbers

- When two signed numbers that are being multiplied have the same sign, multiply their absolute values and make the result positive.

- When two signed numbers to be multiplied have different signs, multiply their absolute values and make the result negative.

- The product of two negative numbers must be positive because the distributive property must hold true for all numbers.

- Use the same procedure when numbers have no signs. Treat unsigned numbers as positive numbers.

- Multiplying any number by –1 results in the opposite of the number.

- When more than two signed numbers are being multiplied, the sign of the answer depends on how many negative numbers are being multiplied.

  Steps

  1. Write the product of all the absolute values.
  2. Count the number of negative numbers that are being multiplied.
  3. Give the result a sign:

     - IF THERE IS AN EVEN NUMBER OF NEGATIVE NUMBERS, make the result positive.

     - IF THERE IS AN ODD NUMBER OF NEGATIVE NUMBERS, make the result negative.

- Use the same procedure to multiply signed fractions or decimal numbers.

## Dividing One Signed Number by Another

- When the numbers to be divided have the same sign, divide their absolute values and make the result positive.
- When the numbers to be divided have different signs, divide their absolute values and make the result negative.
- Treat unsigned numbers as positive numbers.
- Dividing any number by –1 results in the opposite of the number.
- Dividing any number by its opposite results in –1.
- Use the same procedure to divide signed fractions or decimal numbers.

## Commutative, Associative, and Distributive Properties of Operations on Signed Numbers

- The COMMUTATIVE properties of addition and multiplication apply to signed numbers.
- The ASSOCIATIVE properties of addition and multiplication apply to signed numbers.
- Multiplication is DISTRIBUTIVE over addition and subtraction of signed numbers.

## Order of Operations

- When a complex problem includes signed numbers, use the order of operations (PEMDAS) to solve it.

## Exponentials with Negative Bases

- Because we can multiply a negative number by itself several times, it can be written as an exponential that has a negative base.
- Odd powers of negative numbers have negative values. Even powers of negative numbers have positive values.
- The procedures for simplifying exponentials with positive bases also apply to exponentials with negative bases.

## Exponentials with Negative Exponents

- A NEGATIVE EXPONENT results when one exponential is divided by another that has the same base and a larger exponent than the one being divided.
- When an exponent is negative, the value of the exponential is a fraction. The numerator of the fraction is 1 and the denominator is the initial exponential with a positive exponent.
- The procedures for simplifying exponentials that have positive exponents also apply to those that have negative exponents.

# Multiplying and Dividing by Powers of 10 That Have Negative Exponents

- Multiplying a decimal number by a power of 10 that has a negative exponent is the same as dividing the decimal by the exponential with a positive exponent.

- To multiply a decimal number by a power of 10 that has a negative exponent, move the decimal point to the left as many places as the absolute value of the exponent of the power of 10.

- Dividing a decimal number by a power of 10 that has a negative exponent is the same as multiplying the decimal by an exponential with a positive exponent.

- To divide a decimal number by a power of 10 that has a negative exponent, move the decimal point to the right as many places as the absolute value of the exponent of the power of 10.

## Scientific Notation (for Small Numbers)

- Scientific notation is the way scientists write approximations for very small numbers as well as large ones. (See Page 38.)

- Scientific notation is the product of two terms. The first term is a number between 1 and 10. The second term, for very small numbers, is a power of 10 with a negative exponent.

- Any very small number can be written in scientific notation.

  Steps

  1. Ignore the leading zeros and copy the remaining digits.
  2. Place a decimal point after the first digit that you copied.
  3. In the original number, count the digits between where the decimal point had been and where it was moved to.
  4. Use the negative of this number as the exponent in the power of 10.

## Using Exponentials to Identify Place Values

- The place values for decimal numbers are all powers of 10, and can be written using exponentials.

- The ones column is in the center of the place values, just as 0 is in the center of the positive and negative numbers.

| 1000 | 100 | 10 | 1 | | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ |
|---|---|---|---|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | $10^0$ | . | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |

# Algebra
**(Selected Topics)**

## Algebraic Expressions and Equations

- Algebraic expressions and equations use letters to represent numbers.

- An algebraic EQUATION has an equals sign.
  An algebraic EXPRESSION does *not* have an equals sign.

- A number in an algebraic expression or equation is called a CONSTANT.

- A letter in an algebraic expression or equation is called a VARIABLE.

- Algebra avoids the multiplication symbol so that it is not mistaken for the variable “x”. Multiplication is either indicated by parentheses or by placing two letters or a number and a letter next to one another.

## Finding the Value of an Expression

- To find the value of an expression, you must replace all variables with their values, and then find the result.

## General Guidelines for Solving All Equations

- Solve an equation containing a single variable by getting the variable by itself on one side of the equals sign.

- Reverse an operation to remove a term.

- Keep the equation balanced by performing the same operation on both sides of the equals sign.

- Check your answer by placing it in the original equation to verify that the result is true.

# Guidelines for Solving Equations That Have Several Operations

### Begin by moving isolated constants to the side that has no variable

- First, move isolated "+" or "–" constants away from the variable.

### Leave a positive variable in the result

- Do not leave a "–" before a variable in your answer. (If necessary, multiply both sides by –1.)

### Remove variables from the denominator

- Remove a variable from the denominator by multiplying.

### Eliminate parentheses

- Eliminate parentheses by division or distribution.

### Combine similar terms

- Get all similar terms on the same side of the equation and then combine them.

### Eliminate common factors

- Cancel out factors that are common to every group of terms on both sides of an equation.

## Solving a System of Equations

- A SYSTEM OF EQUATIONS is a group of related equations that contain information about two or more variables.
- Solve a system of equations by first finding one of the variables, then substituting its value into the other equation.

## Using Algebra to Solve Percent Problems

- Every percent problem can be written as a proportion:

$$\frac{\textbf{Percent}}{\textbf{100}} = \frac{\textbf{Part}}{\textbf{Whole}}$$

  where one of these three terms will be unknown.

- Since equal fractions have equal cross-products, cross-multiply and solve for the unknown.

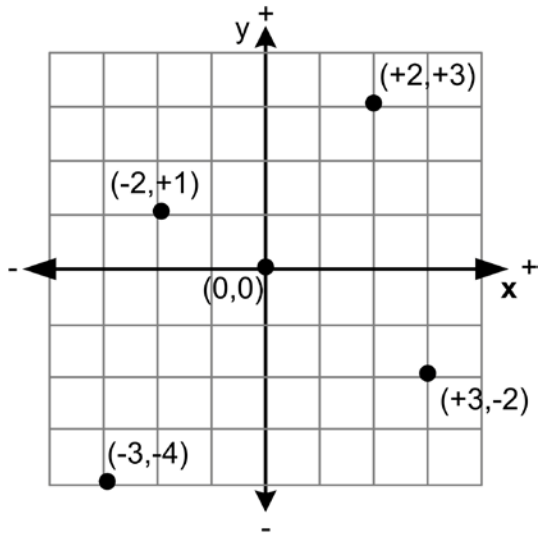
## Multiplying Monomials and Polynomials

- A MONOMIAL is a term made up of the product of a constant and one or more variables.
- Multiply two monomials by multiplying their constants and variables together.
- A POLYNOMIAL is an expression made up of variables and constants, using the operations of addition, subtraction, as well as multiplication.
- Multiply two polynomials column by column, keeping similar terms in the same column.

# Graphing on a Coordinate Grid

## Graphing points on a coordinate grid

- A COORDINATE GRID is made up of evenly spaced horizontal and vertical lines.
- The X-AXIS and Y-AXIS divide the grid into four QUADRANTS.
- Each intersection of the lines is identified by an (x,y) coordinate.
- The intersection of the x and y axes has a coordinate of (0,0) and is called the ORIGIN.
- Graph a point by putting a dot on the grid at its coordinates.




(*Graphing on a Coordinate Grid* Continues)

## Graphing a linear equation

- A LINEAR EQUATION represents the relationship of two variables that is a straight line when graphed.

- A linear equation has the following form:

  $$y = mx + b$$

  where:
  m is the SLOPE of the line, which is the change in y for each increase of x by 1
  b is the Y-INTERCEPT, which is the value of y when x = 0

- To graph a linear equation, select a few values for x and calculate the value of y. Then graph the points on the xy-coordinate grid and connect the dots.

## Graphing a quadratic equation

- A quadratic equation has a cup (parabolic) shape on a graph.

- A quadratic equation has the following form:

  $y = ax^2 + bx + c$

  where a, b, and c are constants.

- To graph a quadratic equation, select several values for x and calculate the value of y. Then graph a few points on the xy-coordinate grid and connect the dots.

# Formulas

- Formulas are algebraic equations that concisely describe a fixed relationship between stable real-world variables.

## Geometry formulas

Circle formulas

| | |
|---|---|
| Diameter | $d = 2r$ |
| Circumference | $c = \pi d = 2\pi r$ |

Perimeter formulas

| | |
|---|---|
| Rectangle | $P = 2(l + w)$ |
| Square | $P = 4s$ |
| Rhombus | $P = 4s$ |
| Equilateral Triangle | $P = 3s$ |

Area formulas

| | |
|---|---|
| Rectangle | $A = lw$ |
| Square | $A = s^2$ |
| Parallelogram/Rhombus | $A = bh$ |
| Triangle | $A = \frac{bh}{2}$ |
| Trapezoid | $A = \frac{h(b1+b2)}{2}$ |
| Circle | $A = \pi r^2$ |

Volume formulas

| | |
|---|---|
| Prism | $V = lwh$ |
| Cube | $V = s^3$ |

*(**Formulas** Continues)*

## The quadratic formula

- The QUADRATIC FORMULA is used to find the roots of a quadratic equation of the form:
  $y = ax^2 + bx + c$

- The ROOTS are the values of x when a parabola intersects the x-axis (y = 0) of a coordinate grid.

- The quadratic formula lets us calculate these roots and avoid graphing the equation.

- The quadratic formula is:
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
  where $\pm$ means “plus or minus”.

## Factorials

- The FACTORIAL of a number is the product of that number, times one less, times one less, … until 1 is reached.

- The symbol for factorial is the exclamation point (!).
  $n! = (n)(n - 1)(n - 2)(n - 3) \ldots (2)(1)$

- Zero factorial is defined to be 1.

*(**Formulas** Continues)*

## Combinations

- A COMBINATION is the number of all possible distinct groups that can be made from a number of items.

- The formula used to calculate a combination is:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

where:

n is the total number of items
r is the number of items in a group

## Permutations

- A PERMUTATION is the number of all possible ordered arrangements that can be made from a group of items. The formula used to calculate a permutation is:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

where:

n is the total number of items
r is the number of items in an arrangement

*(**Formulas** Continues)*

## Arithmetic progressions

- An ARITHMETIC (**ar-ith-**met-**ik**) PROGRESSION is a sequence of numbers that increase by repeatedly adding a constant to the last number.

- The formula used to find the nth number in the sequence $a_1$, $a_2$, $a_3$, …, $a_n$ is:

  $a_n = a_1 + (n - 1)d$

  where:

  n is the position of the nth number

  $a_n$ is the nth number

  $a_1$ is the first number

  d is the common difference between the numbers

- The formula used to find the sum of the first n numbers is:

$$S = \frac{n[2a_1 + (n-1)d]}{2}$$

(***Formulas*** *Continues)*

## Geometric progressions

- A geometric progression is a sequence of numbers that increase by repeatedly multiplying the last number by a constant.

- The formula used to find the nth number in the sequence $a_1$, $a_2$, $a_3$, …, $a_n$ is:

$$a_n = a_1(r^{n-1})$$

where:

n is the position of the nth number

$a_n$ is the nth number

$a_1$ is the first number

r is the multiple by which the numbers increase

# Index

Click a **LETTER** to jump to a page in the Index:

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

… then click a **PAGE NUMBER** to jump to a topic.

## A



## B

## C

## D



## E

## F



## G



## H



## I

## K



## L



## M

## N



## O

## P

## Q



## R

## S



## T

## U



## V



## W



## X



## Y



## Z